

# Seminar

A periodically time-varying minimal realization algorithm based on a matrix chain decomposition

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**Abstract:** The state-space realization of linear systems is one of the most studied subjects in linear systems theory. After the solution of the realization problem for the time-invariant case, particular attention was paid to the case of linear periodic systems (see, e.g. [1]-[8]). Recently, such systems have regained importance, for instance, in the context of coding theory (see [9]), where periodic convolutional encoders play an important role, [10]. The majority of the contributions within this area concern the realization of transfer functions as well as impulse responses, thus excluding the case of input/output linear systems without coprime representations. By the end of the eighties of the last century, Jan C. Willems (see [12]-[13]) suggested an approach (nowadays known as the behavioral approach) that considers a wider class of systems and allows to overcome this drawback. According to this approach, the central object in a system is its behavior which consists of all the signals that satisfy the system laws (also called system trajectories). Consequently, the behavior of a system with an input/output representation that is not coprime, contains more trajectories than the set of input/output signals defined by the system transfer function. Our work takes this fact into account. Based on results already obtained in [11], we address the problem of the realization of linear periodic MIMO (Multi-Input Multi-Output) behaviors. For this purpose, given a periodic behavior, we obtain its lifted time-invariant version and construct a behavioral time-invariant (state-space) realization of the lifted behavior. Finally, we recover a periodic (state-space) realization of the periodic behavior based on the obtained time-invariant realization.

#### References

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